Analysis of error propagation in profile measurement by using stitching

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Abstract

Stitching is a technique which enables longer measurement range of profile from several measured profiles having partly overlapped measurement range. Here, accuracy of the profile obtained by stitching was analyzed and estimated considering that error in each measurement is propagated to error of the stitched profile. As a result, error in the stitched profile was expressed as a function of several measurement parameters and. it can optimize the parameters.

1. Introduction

Stitching is promising for highly precise profilometry with long measurement length such as sub-millimeter alignment of main accelerator for few tens kilometers of length in International Linear Collider project[1], [2]and it also enables nanometer order of profilometry in X-ray mirror having meter order of length. Here, stitching in profilometry was modeled using several measurement parameters and accuracy of the profile obtained through stitching was analyzed considering that error caused in each part measurement is propagated to the stitched profile through stitching.

2. Error analysis in stitching

2.1 Modeling of the stitching

Figure 1 shows a model of n-times of stitching in profilometry investigated in this paper, where f(x) and l express total profile to be measured and its length, $f_i(x)$s and Ls express partly measured profiles from the total profile and their length called unit measurement length, and k*Ls express overlaps of the measurement range, where k is defined as an overlapping ratio against the unit measurement length L.

Figure 2 shows connection scheme of the two measured profiles located next to each other. Here, $f_{i-1}(x)$ and $f_i(x)$ drawn by solid lines express two measured profiles and dotted lines drawn both sides of them express total profile to be measured. The measured profiles are connected as their least square approximation lines, $y=a_{i-1}x+b_{i-1}$ and $y=a_ix+b_i$ match at overlap. Assuming that there is no change in the measured profile except in their slope or offset during measurement, the measured profiles can virtually be extended as shown in Fig. 3 and the relationship between two profiles can be expressed as Eq. 1 for entire measurement position (x=0 to l). In Eq. 1, a_i and b_i stand for their 1st (slope) and 0th (offset) order coefficients of the least square approximation lines, $\Delta a_i=a_i-a_{i-1}$ and $\Delta b_i=b_i-b_{i-1}$ express their difference between the two profiles.

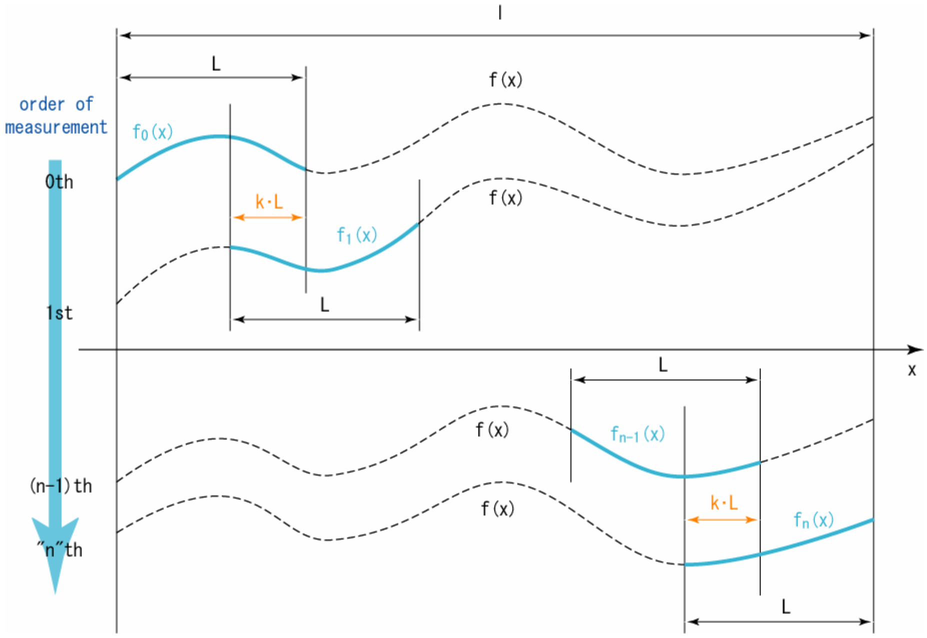

Fig. 1. Model of n-times of stitching in profilometry.

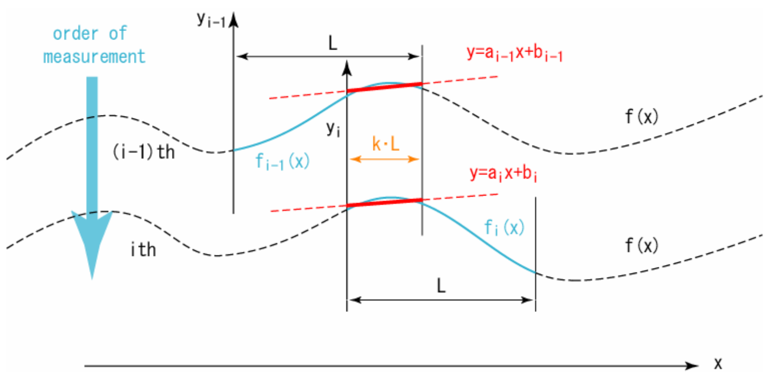

Fig.2. Profile connection using least square approximation lines.

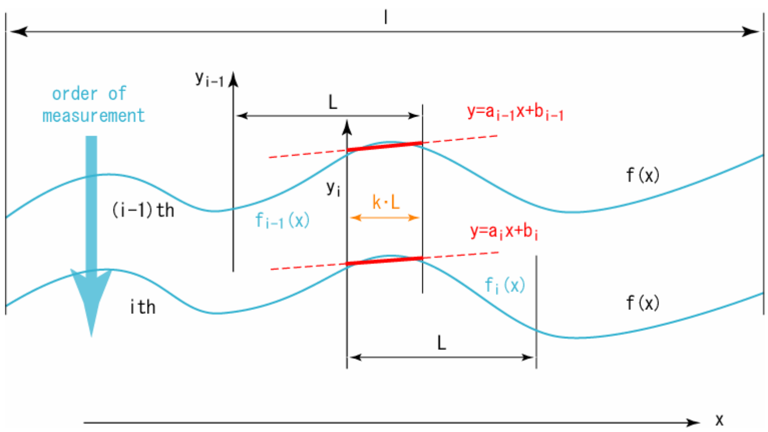

Fig.3. Virtual expansion of the measured profile to the total measurement range (x=0 to l).

$$f_i(x) = f_{i-1}(x) + (\Delta a_i \cdot x + \Delta b_i) \tag{1}$$

Since relationship expressed by Eq. 1 can be applied for all "i"s (i =1 to n), Eqs. 2 is derived as

$$\begin{aligned} f_1(x) &= f_0(x) + (\Delta a_1 \cdot x + \Delta b_1) \\ f_2(x) &= f_1(x) + (\Delta a_2 \cdot x + \Delta b_2) \\ &\vdots \\ f_i(x) &= f_{i-1}(x) + (\Delta a_i \cdot x + \Delta b_i) \\ &\vdots \\ f_{n-1}(x) &= f_{n-2}(x) + (\Delta a_{n-1} \cdot x + \Delta b_{n-1}) \\ f_n(x) &= f_{n-1}(x) + (\Delta a_n \cdot x + \Delta b_n) \end{aligned} \quad . \tag{2}$$

By adding both sides of Eqs. 2, profile through n-times of stitching $f_n(x)$ can be expressed as

$$f_n(x) = f_0(x) + \sum_{i=1}^{n} (\Delta a_i \cdot x + \Delta b_i), \tag{3}$$

where $f_0(x)$ stands for profile without stitching.

The second term of right hand side of Eq. 3 stands for summation of the difference between two least square approximation lines of measured profiles at "n" of overlaps. Thus, profile obtained through stitching can be expressed by profile without stitching and difference between the approximation lines at each overlap.

2.2 Analysis of error propagation toward the approximation line

Fig. 4 shows measured profile and its approximation line at the overlap. Here, profile $f_i(x)$ is measured at m+1 of measurement points p_i (i=0 to m) with sampling interval s and least square approximation line $y=a_ix+b_i$ is derived from the measured value. There is a relationship among the 4 parameters s, k, L and m as

$$m \cdot s = k \cdot L \, . \tag{4}$$

On the other hand, assuming that error is caused by only for y-directional error σ_d at each measurement point, i.e. there is no error for x-direction, errors σ_{ai} and σ_{bi} of coefficients a_i and b_i for the least square approximation lines obtained from measured value at the m+1 of measurement points p_1, …p_j, …,p_{m-1}, -p_m are expressed as

$$\sigma_{a_i} = \sigma_d \cdot \sqrt{\frac{m+1}{D}}, \qquad (5)$$

$$\sigma_{b_i} = \sigma_d \cdot \sqrt{\frac{\sum x^2}{D}}, \qquad (6)$$

respectively. Here, x means x-coordinate of each measurement point and D is expressed as

$$D = (m+1) \cdot \sum x^2 - \left(\sum x\right)^2 . \qquad (7)$$

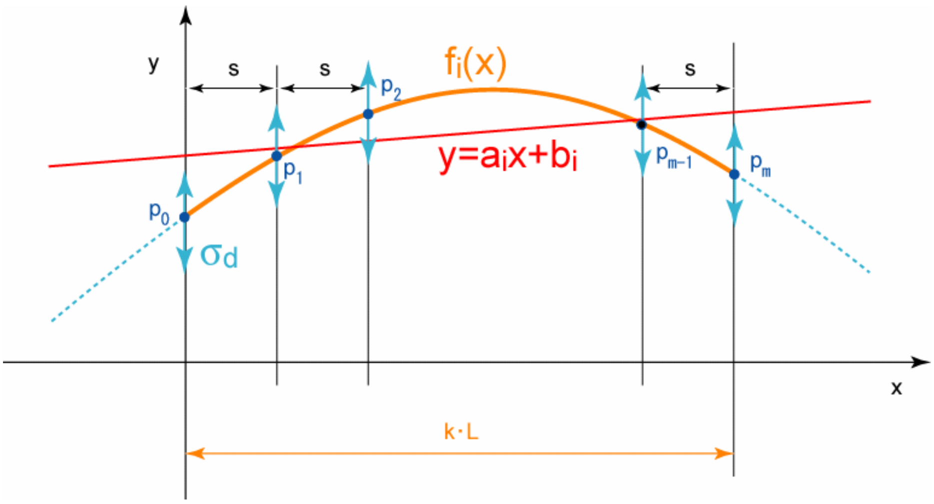

Fig.4. Measured profile and its approximation line at the overlap.
p_i (i=0 to m) express measurement points.

Since x-coordinate for each measurement point is multiple of sampling interval s, Σx^2 and Σx are expressed as

$$\begin{aligned} \sum x^2 &= 0^2 + s^2 + (2s)^2 + \cdots + (m \cdot s)^2 \\ &= s^2 \cdot \left(1^2 + 2^2 + \cdots + m^2\right) \\ &= \frac{s^2}{6} m \cdot (m+1) \cdot (2m+1) \end{aligned}, \qquad (8)$$

$$\begin{aligned} \sum x &= 0 + s + 2s + \cdots + m \cdot s \\ &= s \cdot (1 + 2 + \cdots + m) \\ &= \frac{s}{2} m \cdot (m+1) \end{aligned}, \qquad (9)$$

respectively. Therefore, D is expressed as

$$D = \frac{s^2}{12} m \cdot (m+1)^2 \cdot (m+2) \tag{10}$$

Therefore, σ_{ai}, σ_{bi} are expressed as

$$\sigma_{a_i} = \sigma_d \cdot \sqrt{\frac{12}{s^2 \cdot m \cdot (m+1) \cdot (m+2)}} \quad , \tag{11}$$

$$\sigma_{b_i} = \sigma_d \cdot \sqrt{\frac{2(2m+1)}{(m+1) \cdot (m+2)}} \quad . \tag{12}$$

Here, using relationship shown in Eq. 4, Eqs. 11 and 12 are transformed as

$$\sigma_{a_{i(i=0\sim n)}} = \sigma_d \cdot \sqrt{\frac{12s}{k \cdot L \cdot (k \cdot L + s) \cdot (k \cdot L + 2s)}} \equiv \sigma_a , \tag{13}$$

$$\sigma_{b_{i(i=0\sim n)}} = \sigma_d \cdot \sqrt{\frac{2s \cdot (2k \cdot L + s)}{(k \cdot L + s) \cdot (k \cdot L + 2s)}} \equiv \sigma_b , \tag{14}$$

respectively, since the relationships expressed in Eqs. 11 and 12 are truth for all “i”s, σ_{ai} and σ_{bi} (i=0～n) are expressed as σ_a, σ_b.

2.3 Analysis of error propagation for the stitched profile

Error propagation for stitched profile was estimated by using the value at the end of measurement point, x=l, where the accumulated error by stitching expected to be maximum. Eq. 15 is obtained by substituting l for x in Eq. 3.

$$f_n(l) = f_0(l) + \sum_{i=1}^{n} (\Delta a_i \cdot l + \Delta b_i) \tag{15}$$

From Eq. 15, error at the end of measurement point x=l is expressed by addition of σ_d, error in $f_0(l)$ and σ_s, error in the second term of the right hand side, that is caused by stitching. Since the two errors are independent each other, the total error through stitching is expressed by square root of their squared sum as shown in Eq. 16.

$$\sigma_e = \sqrt{\sigma_d^{\ 2} + \sigma_s^{\ 2}} \qquad (16)$$

Error propagation, $\sigma_{\Delta ai}$, $\sigma_{\Delta bi}$ for the two coefficients in Eq. 15, $\Delta a_i = a_i - a_{i-1}$, $\Delta b_i = b_i - b_{i-1}$ are expressed as

$$\sigma_{\Delta a_{i(i=0\sim n)}} = \sqrt{\sigma_{a_i}^{\ 2} + \sigma_{a_{i-1}}^{\ 2}} = \sqrt{\sigma_a^{\ 2} + \sigma_a^{\ 2}} = \sqrt{2}\sigma_a \fallingdotseq \sigma_{\Delta a} \ , \quad (17)$$

$$\sigma_{\Delta b_{i(i=0\sim n)}} = \sqrt{\sigma_{b_i}^{\ 2} + \sigma_{b_{i-1}}^{\ 2}} = \sqrt{\sigma_b^{\ 2} + \sigma_b^{\ 2}} = \sqrt{2}\sigma_b \fallingdotseq \sigma_{\Delta b} \ , \quad (18)$$

respectively. Here, a_i and a_{i-1}, and b_i and b_{i-1} are respectively independent, as they are derived from deferent measurement value. On the other hand, since coefficients a_i and b_i for the least square approximation line are independent of i, error propagations for their difference, $\sigma_{\Delta ai}$, $\sigma_{\Delta bi}$ are also independent of i.

σ_{sui}, error propagation for $\Delta a_i + \Delta b_i$, which expresses difference between the two approximation lines at the overlapping area is expressed as

$$\sigma_{su_i} = \sigma_{\Delta a} \cdot l + \sigma_{\Delta b} = \sqrt{2} \cdot \left(\sigma_a \cdot l + \sigma_b\right) \fallingdotseq \sigma_{su} \ . \qquad (19)$$

Here, Δa_i and Δb_i are dependent, since they are obtained from the identical measurements. On the other hand, σ_{sui} is independent of i.

As a result, σ_s, error propagation through stitching is expressed as

$$\sigma_s = \sqrt{n} \cdot \sigma_{su} = \sqrt{2 \cdot n} \cdot \left(\sigma_a \cdot l + \sigma_b\right). \qquad (20)$$

since each σ_{su} is independent. Eq. 20 is transformed to

$$\sigma_s = \sqrt{2 \cdot n} \cdot \sqrt{\frac{2s}{(k \cdot L + s) \cdot (k \cdot L + 2s)}} \cdot \left(\sqrt{\frac{6}{k \cdot L}} \cdot l + \sqrt{2k \cdot L + s}\right) \cdot \sigma_d \, , \quad (21)$$

using relationship expressed in Eqs. 13 and 14. Here, number of stitching n is expressed as

$$n = \frac{l - k \cdot L}{L \cdot (1 - k)} \quad (22)$$

Eq. 21 is transformed to

$$\sigma_s = 2 \cdot \sqrt{\frac{s \cdot (l - k \cdot L)}{L \cdot (1 - k) \cdot (k \cdot L + s) \cdot (k \cdot L + 2s)}} \cdot \left(\sqrt{\frac{6}{k \cdot L}} \cdot l + \sqrt{2k \cdot L + s} \right) \cdot \sigma_d \cdot \quad (23)$$

Therefore, the total error propagation through stitching σ_e is expressed as

$$\sigma_e = \sqrt{1 + \frac{4s \cdot (l - k \cdot L)}{L \cdot (1 - k) \cdot (k \cdot L + s) \cdot (k \cdot L + 2s)} \cdot \left(\sqrt{\frac{6}{k \cdot L}} \cdot l + \sqrt{2k \cdot L + s} \right)^2} \cdot \sigma_d \quad (24)$$

using 4 kinds of measurement parameters, l, L, s and k and error in the unit measurement σ_d.

3. Estimation for effects of the measurement parameters

Measurement parameters concerning with measurement length in Eq. 24, l, L, s are made to dimensionless by coefficients u and v as

$$u = \frac{L}{s}, \quad (25)$$

$$v = \frac{l}{L}, \quad (26)$$

respectively. u named sampling coefficient stands for number of sampling point within the unit measurement length. v named measurement length expansion coefficient stands for expansion ratio for measurement length by stitching.

Eq. 24 is transformed using u and v as

$$\sigma_e = \sqrt{1 + \frac{4(v - k)}{(1 - k) \cdot (1 + uk) \cdot (2 + uk)} \cdot \left(\sqrt{\frac{6uv^2}{k}} + \sqrt{2uk + 1} \right)^2} \cdot \sigma_d = K_e \cdot \sigma_d, \quad (27)$$

where K_e expresses magnification of error caused by stitching named error propagation coefficient.

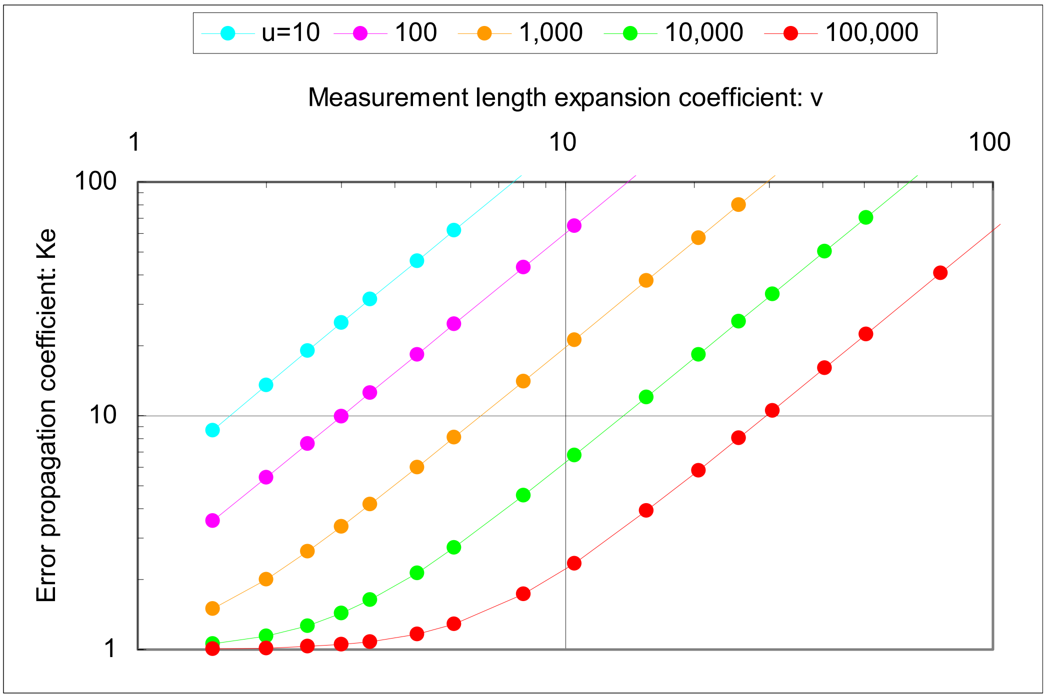

Fig. 5. Error propagation coefficient K_e as a function of measurement length expansion coefficient v using sampling coefficients u as a parameter in case k=0.5

Fig. 5 shows K_e as a function of v in case k=0.5, for u=10, 100, 1000, 10000, 100000 as a parameter, using the relation shown in Eq. 27. Here for example, error in the stitched profile having total measurement length of 10-times of the unit measurement in case using u=1000 and k=0.5 is expressed to be approximately 20-times of the error in the unit measurement.

Figure 6 shows K_e as a function of k in case v=10, for u=10, 100, 1000, 10000, 100000 as a parameter, using the relation shown in Eq. 27. Here for example, error in the stitched profile having total measurement length of 10-times of the unit measurement using u=1000 is expressed to be minimum in case k=0.7 to 0.8 and the error becomes approximately 20-times of the error in the unit measurement.

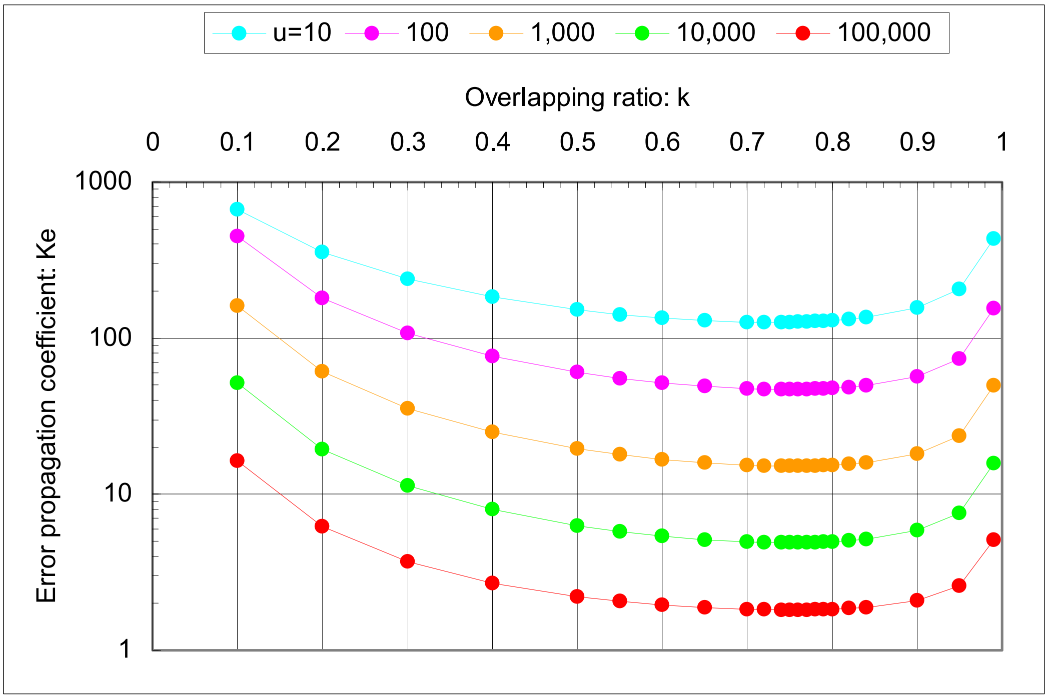

Fig. 6. Error propagation coefficient K_e as a function of overlapping ratio k using sampling coefficients u as a parameter in case v=10.

4. Conclusions

Here, in order to optimize measurement parameters in case using stitching for applying sub-millimeter alignment of main accelerator having few tens kilometers of length in International Linear Collider project.

Error in stitched profile was analyzed assuming that error is caused by each unit measurement and propagated obeying error propagation rule. As a result error propagated toward stitched profile was expressed by 4 measurement parameters (l, L, s, k). The relation was generalized by 3 dimensionless parameters (u, v, k), showing that measurement conditions can be optimized by selecting appropriate combination of the parameters. It shows that this study can also be applied for profilometry for highly precise X-ray mirror (nm accuracy for longer than 100 mm length) not only for alignment of linear collider.

References

[1] http://www.linearcollider.org/cms/.

[2] 2006 spring JSPE biannual meeting N15 (in Japanese).