Analysis of error propagation in profile measurement by using stitching

Tatsuya KUME, Kazuhiro ENAMI, Yasuo HIGASHI, and Kenji UENO

High Energy Accelerator Research Organization (KEK)
1-1 Ōho, Tsukuba, Ibaraki, 305-0801, JAPAN

Stitching is promising for highly precise profilometry with long measurement length such as sub-millimeter alignment in ILC for few tens kilometers of length\(^1\) and nanometer order of profile measurement in X-ray mirror having more than 100 millimeter order of length. Here, accuracy of the profile obtained by stitching is analyzed considering that error in each measurement is propagated to the error in profile.

Figure 1 shows a scheme of a stitching in profilometry, \(f(x)\) and \(l\) express the total profile to be measured and its length, \(f_i(x)\) and \(L\) express partly measured profiles and their length, and \(kL_s\) express overlaps of the measurement range. Considering that profiles next to each other \(f_i(x)\) and \(f_{i+1}(x)\) are connected by using their 1st approximation at the overlapped range, profile through \(n\) times of stitching \(f_n(x)\) can be expressed as equation 1 by the profile without stitching \(f_0(x)\) and differences of the slopes \(\Delta a_i\) and offsets \(\Delta b_i\).

\[
f_n(x) = f_0(x) + \sum_{i=1}^{n} (\Delta a_i \cdot x + \Delta b_i)
\]

(1)

Error in profile obtained by stitching \(\sigma_e\) can be expressed by error in \(f_n(l)\) as equation 2.\(^2\) Here, \(\sigma_d\) stands for error in each measurement, parameters expressing measurement lengths \(l, L\), and sampling interval \(s\) are made to be dimensionless by two coefficients named sampling coefficient \(u = L/s\) and measurement length expansion coefficient \(v = l/L\). \(K_e\) expresses magnification of the profile error caused by stitching and is named an error propagation coefficient. Figure 2 shows \(K_e\) as a function of overlapping ratio \(k\) in case \(n=10\), for \(u=10, 100, 1000, 10000, 100000\) as a parameter.

\[
\sigma_e = \left\{ \frac{4 \cdot (v-k)}{(1-k) \cdot (1+uk) \cdot (2+uk)} \right\} \left( \frac{6uv^2}{k} + \sqrt{2uk + 1} \right)^2 + 1 \cdot \sigma_d = K_e \cdot \sigma_d
\]

(2)